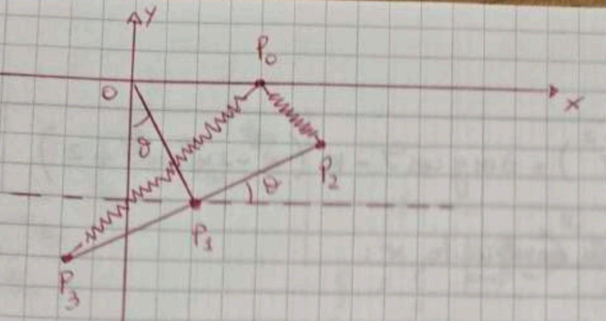


09/06/2025

$P_0 = (x; 0)$

$P_1 = (\sin\vartheta; -\cos\vartheta)$

$P_2 = (\sin\vartheta + \cos\vartheta; -\cos\vartheta + \sin\vartheta)$

$P_3 = (\sin\vartheta - \cos\vartheta; -\cos\vartheta - \sin\vartheta)$

$\vec{V}_0 = (\dot{x}; 0)$

$\vec{V}_1 = (\dot{\vartheta}\cos\vartheta; \dot{\vartheta}\sin\vartheta)$

$\vec{V}_2 = (\dot{\vartheta}\cos\vartheta - \dot{\vartheta}\sin\vartheta, \dot{\vartheta}\sin\vartheta + \dot{\vartheta}\cos\vartheta)$

$\vec{V}_2 = (\dot{\vartheta}\cos\vartheta + \dot{\vartheta}\sin\vartheta; \dot{\vartheta}\sin\vartheta - \dot{\vartheta}\cos\vartheta)$

$$T = \frac{1}{2}m\left(v_0^2 + v_1^2 + v_2^2 + v_3^2\right) =$$

$$= \frac{1}{2}m\Big(\dot{x}^2 + \dot{\vartheta}^2\cos^2\vartheta + \dot{\vartheta}^2\sin^2\vartheta + \dot{\vartheta}^2\cos^2\vartheta + \dot{\vartheta}^2\sin^2\vartheta - 2\dot{\vartheta}^2\cancel{\sin\vartheta\cos\vartheta} +$$
$$+ \dot{\vartheta}^2\sin^2\vartheta + \dot{\vartheta}^2\cos^2\vartheta + 2\dot{\vartheta}^2\cancel{\sin\vartheta\cos\vartheta} + \dot{\vartheta}^2\cos^2\vartheta + \dot{\vartheta}^2\sin^2\vartheta + 2\dot{\vartheta}^2\cancel{\sin\vartheta\cos\vartheta} +$$
$$+ \dot{\vartheta}^2\sin^2\vartheta + \dot{\vartheta}^2\cos^2\vartheta - 2\dot{\vartheta}^2\cancel{\sin\vartheta\cos\vartheta}\Big) =$$

$$= \frac{1}{2}m\left(\dot{x}^2 + 5\dot{\vartheta}^2\right)$$

Il potenziale del sistema è:

$$U = U_1 + U_2 + U_3 + U_4 + U_{ee_1} + U_{ee_2} =$$

$$= -mgy_0 - mgy_1 - mgy_2 - mgy_3 - \frac{1}{2}k\left(|\vec{P_0P_2}|^2 + |\vec{P_0P_3}|^2\right) =$$

$$= mg\cos\vartheta + mg\cos\vartheta - mg\cancel{\sin\vartheta} + mg\cos\vartheta + mg\cancel{\sin\vartheta} - \frac{1}{2}k\Big((x - \sin\vartheta - \cos\vartheta)^2 +$$
$$+ (\cos\vartheta - \sin\vartheta)^2 + (x - \sin\vartheta + \cos\vartheta)^2 + (\cos\vartheta + \sin\vartheta)^2\Big) =$$

$$= 3mg\cos\vartheta - \frac{1}{2}k\Big[x^2 + \sin^2\vartheta + \cos^2\vartheta - 2x\sin\vartheta - \cancel{2x\cos\vartheta} + 2\cancel{\sin\vartheta\cos\vartheta} + \cos^2\vartheta + \sin^2\vartheta$$
$$- 2\cancel{\sin\vartheta\cos\vartheta} + x^2 + \sin^2\vartheta + \cos^2\vartheta - 2x\sin\vartheta + \cancel{2x\cos\vartheta} - 2\cancel{\sin\vartheta\cos\vartheta} + \cos^2\vartheta + \sin^2\vartheta$$
$$+ 2\cancel{\sin\vartheta\cos\vartheta}\Big] =$$

$$= 3mg\cos\vartheta - \frac{1}{2}k\left[2x^2 + 4\sin^2\vartheta + 4\cos^2\vartheta - 4x\sin\vartheta\right] =$$

$$= 3mg\cos\vartheta - k\left[x^2 + 2 - 2x\sin\vartheta\right]$$

La Lagrangiana è quindi:

$$\boxed{L(x,\vartheta;\dot{x},\dot{\vartheta}) = \frac{1}{2}m\left(\dot{x}^2+5\dot{\vartheta}^2\right)+3mg\cos\vartheta-K\left(x^2-2x\sin\vartheta+2\right)}$$

Troviamo l'equazione di Lagrange per la coordinata x:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=0 \Rightarrow$$

$$\left.\begin{array}{l} \dfrac{\partial L}{\partial \dot{x}} = m\dot{x} \Rightarrow \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}}\right)=m\ddot{x} \\ \dfrac{\partial L}{\partial x} = -2Kx+2K\sin\vartheta \end{array}\right\} \Rightarrow \boxed{m\ddot{x}+2Kx-2K\sin\vartheta=0}$$

Troviamo l'equazione di Lagrange per la coordinata ϑ:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\vartheta}}\right)-\frac{\partial L}{\partial \vartheta}=0 \Rightarrow$$

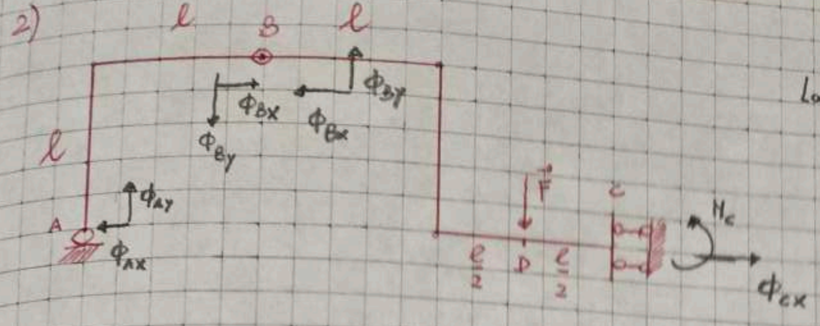
$$\left.\begin{array}{l} \dfrac{\partial L}{\partial \dot{\vartheta}} = 5m\dot{\vartheta} \Rightarrow \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{\vartheta}}\right)=5m\ddot{\vartheta} \\ \dfrac{\partial L}{\partial \vartheta} = -3mg\sin\vartheta+2Kx\cos\vartheta \end{array}\right\} \Rightarrow \boxed{5m\ddot{\vartheta}+3mg\sin\vartheta-2Kx\cos\vartheta=0}$$

Per quanto riguarda le posizioni di equilibrio si ha:

$$\begin{cases} \dfrac{\partial U}{\partial x}=0 \Rightarrow \\ \dfrac{\partial U}{\partial \vartheta}=0 \Rightarrow \end{cases} \begin{cases} -2Kx+2K\sin\vartheta=0 \\ -3mg\sin\vartheta+2Kx\cos\vartheta=0 \end{cases} \Rightarrow \begin{cases} x=\sin\vartheta \\ \sin\vartheta\left(-3mg+2Kx\cos\vartheta\right)=0 \end{cases} \Rightarrow$$

$$\boxed{\begin{array}{l} A_1=(0;0),\quad A_2=(0;\pi),\quad A_3=\left(\sqrt{1-\dfrac{9m^2g^2}{4K^2}}\,;\ \arccos\dfrac{3mg}{2K}\right), \\ A_4=\left(-\sqrt{1-\dfrac{9m^2g^2}{4K^2}}\,;\ -\arccos\dfrac{3mg}{2K}\right) \end{array}}$$

2)



La struttura è isostatica.

Scriviamo le equazioni cardinali per il tratto AB:

$$\begin{cases} -\phi_{Ax} + \phi_{Bx} = 0 \\ \phi_{Ay} - \phi_{By} = 0 \\ (A-B) \times \vec{\phi}_A = 0 \quad (\text{polo in } B) \end{cases} \Rightarrow \begin{cases} \phi_{Ax} = \phi_{Bx} \\ \phi_{Ay} = \phi_{By} \\ (-\ell \underline{i} - \ell \underline{j}) \times (-\phi_{Ax} \underline{i} + \phi_{Ay} \underline{j}) = 0 \end{cases} \Rightarrow$$

$$\begin{cases} \phi_{Ax} = \phi_{Bx} \\ \phi_{Ay} = \phi_{By} \\ -\ell \phi_{Ay} - \ell \phi_{Ax} = 0 \end{cases} \Rightarrow \begin{cases} \phi_{Ax} = \phi_{Bx} \\ \phi_{Ay} = \phi_{By} \\ \phi_{Ax} = -\phi_{Ay} \end{cases}$$

Per il tratto BC si ha:

$$\begin{cases} -\phi_{Bx} + \phi_{cx} = 0 \\ \phi_{By} - F = 0 \\ (D-B) \times \vec{F} + \vec{M}_c + (C-B) \times \vec{\phi}_c = 0 \quad (\text{polo in } B) \end{cases} \Rightarrow \begin{cases} \phi_{Bx} = \phi_{cx} \\ \phi_{By} = F \\ \left(\frac{3}{2}\ell \underline{i} - \ell \underline{j}\right) \times \left(-F \underline{j}\right) + M_c \underline{k} + \\ \quad + (2\ell \underline{i} - \ell \underline{j}) \times (\phi_{cx} \underline{i}) = 0 \end{cases} \Rightarrow$$

$$\begin{cases} \phi_{Bx} = \phi_{cx} \\ \phi_{By} = F \\ -\frac{3}{2} \ell F + M_c + \ell \phi_{cx} = 0 \end{cases} \Rightarrow \boxed{\begin{cases} \vec{\phi}_A = (-F; F) \\ \vec{\phi}_B = (-F; F) \\ \vec{\phi}_c = (-F; 0) \\ M_c = \frac{5}{2} F \ell \end{cases}}$$